THE TRANSFER OF HEAT FROM A ROTATING DISK TO A LIQUID IN A REGIME OF LAMINAR FLOW

P. L. Zarubin and L. A. Poluboyartseva

Inzhenerno-Fizicheskii Zhurnal, Vol. 14, No. 3, pp. 558–563, 1968

UDC 536.242

We present the results from an experimental study of the transfer of heat from a disk for a substantial temperature difference between the liquid and the disk, and for a change in the viscosity of the liquid over a wide range.

The problem of the transfer of heat from a rotating disk in a regime of laminar flow has been dealt with by many authors [1–14]. Without considering the influence of viscous dissipation on the temperature distribution, the solution of the thermal and diffusion problems is identical. If the temperature difference between the liquid and the disk is not too great, i.e., we can neglect the change in density, viscosity, and thermal conductivity with a change in temperature, the quantity of heat given up by the disk is expressed by the equation [3]

$$Q = \pi a^2 \lambda \omega^{1/2} \nu^{-1/2} t_1'(0) \Delta t, \qquad (1)$$

where

$$t_1'(0) = -\left[\int_0^\infty \exp\left(\mathrm{Pr}_t \int_0^\xi h d\xi\right) d\xi\right]^{-1}. \qquad (2)$$

The integral in (2), solved numerically, has been calculated by many researchers. Kibel has given an exact solution for $\mathrm{Pr}_t = 1$; Millsaps and Pohlhausen have given an exact solution for values up to $\mathrm{Pr}_t = 10$. Dorfman [14] derived values of the function for certain Pr_t numbers in the interval from 0.1 to 100. Sparrow and Gregg carried out the calculation for values of $\mathrm{Pr}_t = 0.01$, 0.1, and 1–100, and they also presented approximate expressions for very small and very large values of Pr_t:

$$t_1'(0) = 0.88447\,\mathrm{Pr}_t \text{ as } \mathrm{Pr}_t \to 0, \qquad (3)$$

$$t_1'(0) = 0.62048\,\mathrm{Pr}_t^{1/3} \text{ as } \mathrm{Pr}_t \to \infty. \qquad (4)$$

Cess gave the values of $t_1'(0)$ for $\mathrm{Pr}_t = 0.02$ and 0.04. Davis calculated the integral for a number of values of Pr_t in the interval from 1 to 1000 and gave the approximate equation

$$t_1'(0) = -\frac{1}{\pi} k \mathrm{Pr}_t^{\frac{1}{2+\alpha}}, \qquad (5)$$

where k is a function of α [3, 7].

When $\alpha = 1$, the satisfactory agreement with the exact value is found for the case in which $\mathrm{Pr}_t > 100$, while when $\alpha = 0.67$, the agreement is found in the interval $6 < \mathrm{Pr}_t < 500$. Approximate have been presented in a number of other references [8–11].

At the Computer Center of the Urals State University, the values of the function $t_1'(0)$ were calculated for values up to $\mathrm{Pr}_t = 50$ (Table 1). In the region of values $\mathrm{Pr}_t > 5$ the function is expressed with satisfactory accuracy by the equation [15]

$$t_1'(0) = 0.468\,\mathrm{Pr}_t^{0.39}. \qquad (6)$$

Having substituted (6) into (1), we obtain

$$Q = 0.468\pi a^2 \lambda \omega^{1/2} \nu^{-1/2} \mathrm{Pr}_t^{0.39} \Delta t. \qquad (7)$$

(Equations (6) and (7) in [15] have been erroneously presented with the coefficient 0.486.)

The experimental data on heat transfer of a rotating disk are few in number and available only for air media [12, 13].

Earlier we investigated the transfer of heat from a disk to water and to a solution of ferric chloride for a temperature difference of 20° between the disk and the liquid [15]. The resulting data corresponded to Eq. (1). A change in the direction of the heat flow and the imposition of diffusion exerted no significant influence on the coefficient of heat transfer. In viscous liquids, given a large temperature difference, we would expect that such an effect would be noticeable, since from

Table 1

The Value of the Function $t_1'(0)$ as a Function of Pr_t

Pr_t	$t_1'(0)$	Pr_t	$t_1'(0)$	Pr_t	$t_1'(0)$	Pr_t	$t_1'(0)$	Pr_t	$t_1'(0)$
1	−0.397	11	−1.179	21	−1.515	31	−1.756	41	−1.948
2	−0.566	12	−1.220	22	−1.543	32	−1.777	42	−1.966
3	−0.683	13	−1.259	23	−1.569	33	−1.797	43	−1.983
4	−0.776	14	−1.296	24	−1.594	34	−1.817	44	−2.000
5	−0.854	15	−1.331	25	−1.619	35	−1.837	45	−2.016
6	−0.922	16	−1.365	26	−1.643	36	−1.856	46	−2.033
7	−0.982	17	−1.398	27	−1.667	37	−1.875	47	−2.049
8	−1.038	18	−1.429	28	−1.690	38	−1.894	48	−2.065
9	−1.088	19	−1.459	29	−1.712	39	−1.912	49	−2.081
10	−1.135	20	−1.488	30	−1.734	40	−1.930	50	−2.096

Table 2

Thermophysical Properties of the Liquids Used in Heat Measurements on a Disk

Liquid	Temperature, °C	Dynamic viscosity, N · sec/m² · 10^{-3}	Density, kg/m³ · 10^{-3}	Kinematic viscosity, m²/sec · 10^{-6}	Heat capacity, J/(kg · deg) · 10^3	Thermal conductivity, W/m · deg	Prandtl number
96% solution of H_2SO_4	25	19.96—24,5	1.83	10.9—13.4	1.34—1.5	0.27—0.33	81.5—137
	60	6.32— 8,5	1.796	3.52— 4.73	1.5	0.34—0.42	22,5—37.7
50% solution of glycerin	25	5.1	1.124	4.54	3.4	0.424	41
	47.5	2.65	1.113	2.38	3.36	0.437	20.4
	70	1.42	1.103	1.29	3.31	0.455	10.34
98.5% solution of glycerin	15	2030	1.26	1610	2.33	0.288	16410
	42.5	251	1.243	202	2.44	0.293	1950
	70	50.8	1.226	41.5	2.46	0.297	419

Table 3

Relative Deviations of $t_1'(0)$ from the Exact Values According to Approximation Equations

Values of $t_1'(0)$	Δ%, for Pr_T					
	1	6	10	100	500	1000
According to Eq. (7)	15.2	1.4	0.4	2.1	6.6	9.5
According to Eq. (5), α = 0.67	41.7	3.8	2.2	1.1	5.5	7.2
According to Eq. (5), α = 1.0	27.8	18.3	15.1	5.6	1.8	1.1
According to Eq. (4)	27.8	18.4	15.2	5.9	1.4	1.45

Table 4

Maximum Relative Deviations of $t_1'(0)$ from the Exact Values on the Basis of Experimental Data

	Heat transfer							
	to water		to a 50% solution of clycerin		to a 96% solution of sulfuric acid		to a 98.5% solution of glycerin	
	I	II	I	II	I	II	I	II
Δ_1, %	+21.1 — 8.7	+7.2 —9.0	+19 —6.6	+9.8 —9.5	+41 —35	+65.4 —29.9	+18.3 — 7.5	+30.4 — 8.7

I) Cooling; II) heating

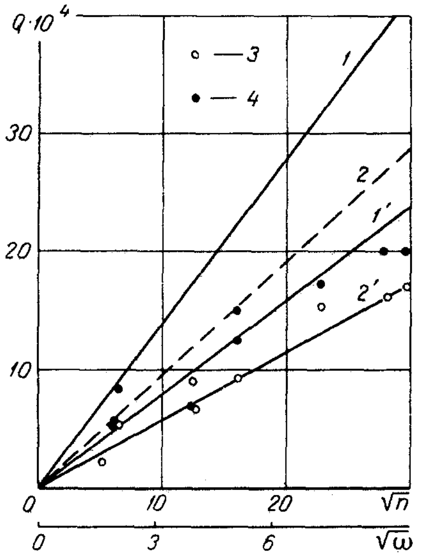

Fig. 1. Heat transfer from disks of steel 20 ($Wt/m^2 \cdot deg$) to 96 percent acid (n, rpm; ω, rad/sec): 1 and 1') heat flux from Eq. (7) at 60°C; 2 and 2') the same at 25°C; 3) t_l = 60°C and t_d = 25°C; 4) t_l = 25°C and t_d = 60°C.

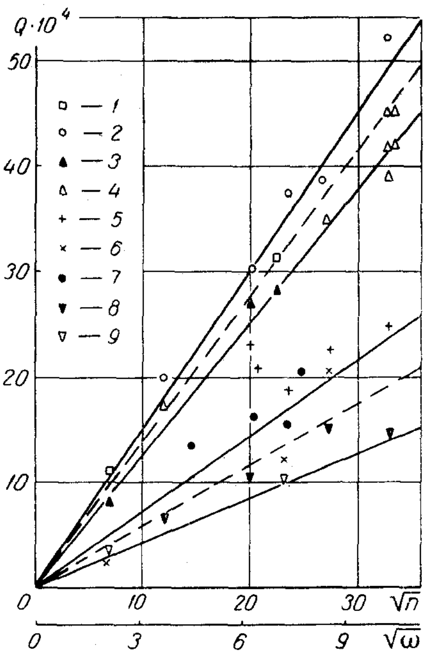

Fig. 2. Heat transfer from copper disks to 50 percent (1-4) and 98.5 percent (5-9) glycerine (n, rpm; ω, rad/sec): 1, 2) t_d = 70°C, t_l = 25°C; 3, 4) t_d = 25°C, t_l = 70°C; 5, 6, 7) t_d = 70°C, t_l = 25°C; 8, 9) t_d = 15°C, t_l = 70°C; 2, 4, 5, 8) temperature recordings at three points; 7) the same with smaller liquid volume.

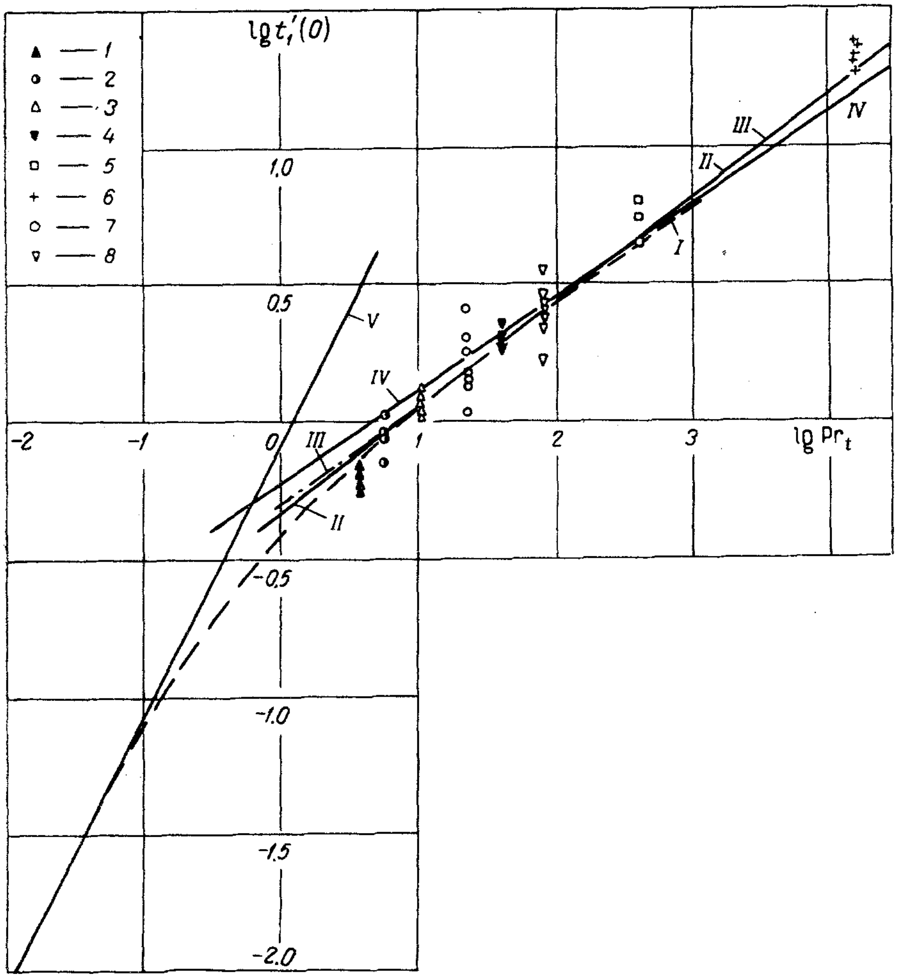

Fig. 3. Heat transfer to liquid according to calculation I) accurate values of the function $t_1'(0)$; II) according to Eq. (7); III) according to Eq. (5), α = 0.67; IV) according to Eqs. (4) and (5), α = 1.0; V) according to Eq. (3) and experimental data for water; 1) t_d = 50°C, t_l = 30°C, Pr_T = 3.6; 2) t_d = 30°C, t_l = 50°C, Pr_T = 5.4; for 50 percent glycerine; 3) t_d = 70°C, t_l = 25°C, Pr_t 10.34; 4) t_d = 25°C; t_l = 70°C, Pr_T = 41; for 98.5 percent glycerine 5) t_d = 70°C, t_l = 15°C, Pr_T = 419; 6) t_d = 15°C, t_l = 70°C, Pr_T = 16410; for 96 percent H_2SO_4 7) t_d = 60°C, t_l = 25°C, Pr_T = 22.7; 8) t_d = 25°C, t_l = 60°C, Pr_T = 81.5.

among all the properties of a liquid it is viscosity that changes most markedly with temperature [16].

To study the transfer of heat from a disk to a viscous liquid we chose a 96% solution of H_2SO_4, and 50% and 98% solutions of glycerin, whose thermophysical properties are given in Table 2. The construction of the instrument and the method for the measurement of the heat flow were described earlier in [15].

Because of the great scatter in the handbook data on the properties of sulfuric acid, the straight lines in Fig. 1 show the values of the heat flow calculated from Eq. (7) with the extremal values for ν, λ, Pr_t, etc. The experimental points lie near the straight lines which correspond to the minimum values for the parameters of the liquid. We note the influence of the heat-flow direction: the transfer of heat from heated disks is greater than from those that are cooled.

The results from experiments on the transfer of heat to glycerin also suggest that the direction of the heat flow affects its magnitude (Fig. 2). The experimental points lie closer to the continuous lines that represent the heat flow as a function of the velocity of disk rotation, when the parameters of the liquid that correspond to the disk temperature are substituted into Eq. (7). The broken lines are plotted for the temperature which represents the mean between the temperatures of the liquid and the disk.

The greatest scattering of data was found in experiments involving the heating of a 98.5% solution of glycerin, probably due to the weak mixing of the viscous solution by the rotating disk. In order to improve the mixing, a smaller quantity (smaller by 30%) of glycerin was taken in some of the experiments. The glycerin temperature was subsequently measured at three points that exhibited the greatest difference with respect to mixing conditions, and we calculated the average heat flow.

Figure 3 shows the experimental data in the coordinates $\lg t_l'(0) - \lg Pr_t$. We calculated the function $t_l'(0)$ from Eq. (1). The temperature of the disk was assumed to be decisive. The broken line corresponds to the exact values of the function $t_l'(0)$; the straight lines were derived from the approximate equations (3)–(5) and (7). Table 3 gives the relative deviations of $t_l'(0)$ from the exact values on the basis of these approximate equations. The approximate equations of Sparrow and Gregg (4) and of Davis (5) for $\alpha = 1$ are virtually coincident. The expression for $t_l'(0)$ derived from the Sundheim [11] equation also coincide with the above-mentioned equations. The Davis equation for $\alpha = 0.67$ is close to Eq. (7).

Table 4 shows the relative deviations of $t_l'(0)$ from the exact values on the basis of experimental results. In the case of the 98.5% solution of glycerin, we carried out a comparison of the values for $t_l'(0)$ derived from Eq. (7). The greatest scatter was found in the case of heat transfer to sulfuric acid to a 98.5% solution of glycerin (heating) i. e., 30–65%. In the remaining cases, the deviations ranged from 6.6 to 21%. Over the entire investigated range of values for Pr_t, the experimental scatter encompasses the difference between the exact and approximate values of $t_l'(0)$.

The derived results permit us to draw the following conclusions. The heat flow from the disk can be calculated theoretically in the case of a substantial temperature difference between the liquid and the disk and if there is a change in the viscosity of the liquid over a wide range. The imposition of diffusion onto the heat-transfer process exerts no significant influence. For liquids whose viscosity is close to that of water, the direction of the heat flow need not be taken into consideration. With an increase in the viscosity, the affect of the heat-flow direction becomes more pronounced and it becomes necessary to take it into consideration in the calculations, which can be done by substituting the values of the parameters of the liquid at the temperature of the disk into the equations.

NOTATION

Q is the heat flux; a is the disk radius; λ and ν are the thermal conductivity and kinematic viscosity of the liquid; ω is the angular velocity of disk rotation; Δt is the temperature difference between disk and liquid; Pr_t is the Prandtl number; Δ is the relative deviation of the function $t_l'(0)$ from accurate values in approximate equations, %; Δ_l is the relative deviation of $t_l'(0)$ in experimental data from accurate values, %; t_d is the disk temperature; t_l liquid temperature.

REFERENCES

1. I. A. Kibel, Prikladnaya matematika i mekhanika, vol. XI, no. 6, 1947.
2. K. Millsaps and K. Pohlhausen, J. Aeronautic. Sci., 19, no. 2, 1952.
3. L. G. Loitsyanskii, The Laminar Boundary Layer [in Russian], Fizmatgiz, 1962.
4. V. G. Levich, Physicochemical Hydrodynamics [in Russian], Fizmatgiz, 1959.
5. E. M. Sparrow and I. L. Greeg, Trans. ASME, series C, 81, 249–251, 1959.
6. R. D. Cess, Applied Scientific Research, sect. A, 13, no. 1, 1964.
7. D. R. Davis, Quart. Journ. Mech. and Applied Math., vol. XII, pt. 1, 14–21, 1959.
8. L. A. Dorfman, Hydrodynamic Resistance and the Heat Transfer of Rotating Bodies [in Russian], Fizmatgiz, 1960.
9. C. J. Wagner, Appl. Phys., 19, 837, 1948.
10. A. C. Oudart, R. Acad. Sci., Paris, 239, no. 1, 1954.
11. B. R. Sundheim and W. Sauerwein, J. Phys. Chem., 69, no. 11, 1965.
12. R. L. Joung, Trans. ASME, 78, 16, 1956.
13. E. C. Cobb and O. A. Saunders, Proc. of the Royal Soc., sec. A, Mathematics and Physical Sci., 236, 343–351, 1956.
14. L. A. Dorfman, A. Z. Serazetdinov, Int. J. Heat Mass Transfer, 8, 317–327, 1965.
15. P. I. Zarubin, L. A. Poluboyartseva, and V. M. Novakovskii, Zashch. met., 1, no. 3, 1965.
16. B. S. Petukhov and E. A. Krasnoshchekov, collection: Heat Transfer and Thermal Modeling [in Russian], Izd. AN SSSR, 165, 1959.

20 June 1967